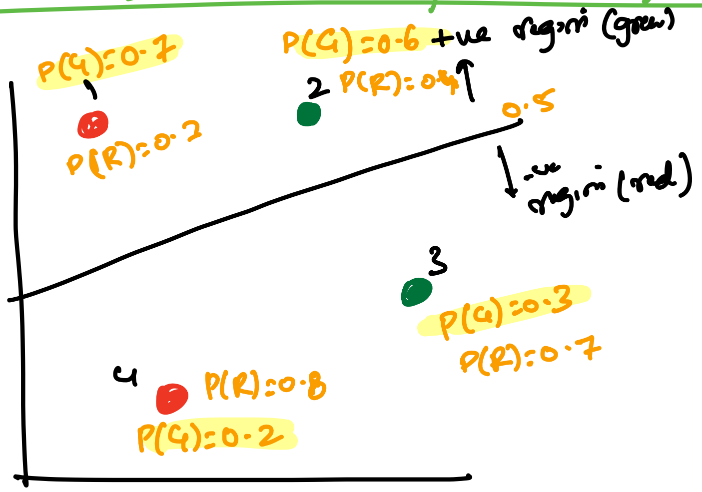
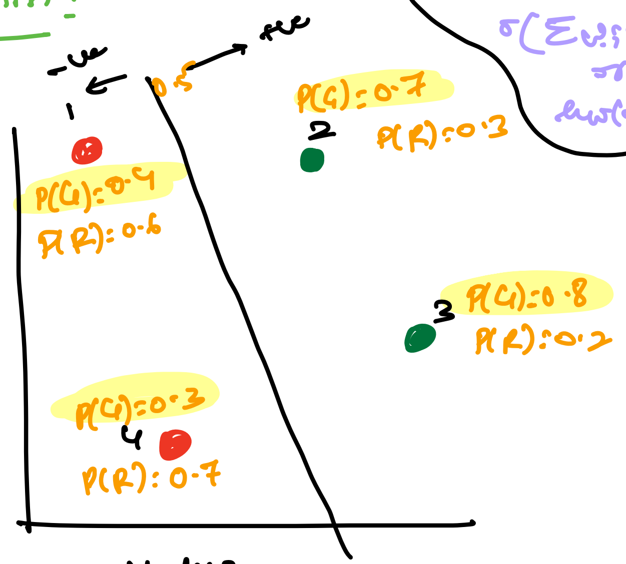
Objective fxn for Logistic Regression:

$\sigma(w^Tx)$ or $\sigma(\sum w_i x_i)$ or $f_w(x)$



Model 1



Model 2

Likelihood $= 0.3 * 0.6 * 0.3 * 0.8$ $\qquad$ $0.6 * 0.7 * 0.8 * 0.7$

$= 0.0432$ $\qquad$ $= 0.2352$ better

4 points $\longrightarrow$ 10,000 points

$\underbrace{\_ * \_ * \_ * \_ \text{———————} * \_}_{10,000}$ = very small

$\log(ab) = \log a + \log b$

log likelihood $= \underset{-ve}{\underline{\log 0.3}} + \underset{-ve}{\underline{\log 0.6}} + \underset{-ve}{\underline{\log 0.3}} + \underset{-ve}{\underline{\log 0.8}}$ $\qquad$ $\log 0.6 + \log 0.7 + \log 0.8 + \log 0.7$

max log-likelihood

negative log likelihood

$= -\log 0.3 - \log 0.6 - \log 0.3 - \log 0.8$

minimize $\underbrace{(\text{negative log likelihood})}_{\text{cross entropy}}$ → cross entropy

|| loss fxn for logistic regression

$$h_w(x) = \sigma\Big(\underbrace{\sum_{i=1}^{n} w_i x_i}_{z}\Big)$$

1 $\longrightarrow 1-h_w(x)$ | $y=0$

2 $\longrightarrow h_w(x)$ | $y=1$

3 $\longrightarrow h_w(x)$ | $y=1$

4 $\longrightarrow 1-h_w(x)$ | $y=0$

$$y.\big(h_w(x)\big) + (1-y)\big(1-h_w(x)\big)$$

$$= -\log 0.3 - \log 0.6 - \log 0.3 - \log 0.8$$

$$= -y^i \log\big(h_w(x^i)\big) - (1-y^i)\log\big(1-h_w(x^i)\big)$$

$$= -\frac{1}{m}\left(\sum_{i=1}^{m} y^i \log\big(h_w(x^i)\big) + (1-y^i)\log\big(1-h_w(x^i)\big)\right)$$

$$= \frac{-1}{m}\left(\sum_{i=1}^{m} y^i \log(\sigma(z)) + (1-y^i)\log(1-\sigma(z))\right)$$

$$z = w^T x$$

final loss fxn

→ Binary cross entropy

→ log-loss error

prioritized? Justify your ...

3. A financial services company is building a logistic regression model to predict whether a customer is likely to default on a loan (outcome: y=1, default, y=0, no default) based on the following features: Income (x_1, in thousands of ₹), Credit Score (x_2), and Number of Dependents (x_3). The coefficients of a trained model are given as $w_0 = 1.0, \mathbf{w} = [-0.02, 0.03, -0.1]^T$. The company uses a threshold of $P(y = 1) = 0.5$ to o classify customers as default or not.

[a] Consider a customer with Income = ₹50,000, Credit Score = 700, and Number of Dependents = 2. Will this customer be classified as likely to default or not? 3 ~~[2]~~ [CO4]

w_0	w_1	w_2	w_3
1.0	-0.02	0.03	-0.1

$\Sigma w_i x_i = z = w_0 + w_1 x_1 + w_2 x_2 + w_3 x_3$

$\Sigma w_i x_i = z = 1.0 - 0.02 * 50{,}000 + 0.03 * 700 - 0.1 * 2 = -978.2$

$\sigma(z) = \sigma(\Sigma w_i x_i) = \dfrac{1}{1+e^{-\Sigma w_i x_i}} = \dfrac{1}{1+e^{978.2}} = \dfrac{1}{1+(2.72)^{978.2}}$

∞

$= \sim 0$

$\sigma(z) \geq 0.5 \rightarrow y = 1$

$\sigma(z) < 0.5 \rightarrow y = 0$

$\sigma(z) \rightarrow \sim 0 \rightarrow y = 0$ (no default)

A logistic regression (LR) model is used to predict whether a student will pass an exam (Pass = 1) or fail (Pass = 0) based on two features: Hours Studied (X_1) and Number of Practice Tests Taken (X_2). The coefficients of trained LR model are: $w_0 = -4$, $w_1 = 0.6$, $w_2 = 0.8$. For a student who studied for 10 hours and took 5 practice tests, determine whether the student will pass or fail based on the probability $P(Pass = 1)$ and a decision threshold of 0.5. What would the classification be if the decision threshold is increased to 0.7? **[4] [CO3, CO4]**

$w_0 = -4 \quad w_1 = 0.6 \quad w_2 = 0.8$

$z = w_0 + w_1 x_1 + w_2 x_2 = -4 + 0.6 * 10 + 0.8 * 5 = -4 + 6 + 4 = 6$

$\sigma(z) = \frac{1}{1+e^{-6}} = \frac{1}{1+\frac{1}{e^6}} = \frac{1}{1+\frac{1}{403.4}} = \frac{1}{1+0.002} = 0.998$

1. $0.998 > 0.5 \Rightarrow y = 1$: Pass
2. $0.998 > 0.7 \Rightarrow y = 1$: Pass